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On a new subclass of Bi-Univalent Functions involving modified Bazilevic structure, Salagean differential Operator and generalized m-leaf Function.

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Abstract

In this paper, we introduce and investigate a new subclass of bi-univalent functions defined via a modified Bazilevic-type structure involving the Salagean differential operator and a generalized m-leaf function. By employing the theory of subordination and Caratheodory functions, we obtain sharp coefficient bounds for the initial Taylor's Maclaurin coefficients. Furthermore, a refined Fekete-Szego inequality is established. Several known results are recovered as special cases of our main results, thereby demonstrating the unifying nature and novelty of the proposed class.

Keywords: Salagean operator, Bi-univalent, M-leaf, Three-leaf

1 Introduction

Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ denote the open unit disk. Let $\mathcal{A}$ be the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in $\mathbb{D}$.

A function $f \in \mathcal{A}$ is said to be *univalent* if it is one-to-one in $\mathbb{D}$. The class of bi-univalent functions Σ consists of functions $f \in \mathcal{A}$ such that both f and its inverse f^{-1} are univalent in $\mathbb{D}$. The study of coefficient bounds for bi-univalent functions was revived by Lewin, (1967), and later developed extensively by Brannan,(1985), Netanyahu,(1969),Srivastava,(2014), Bulut,(2013), Ali, (2012), Frasin (2011), Caglar,(2012).

Operator-based generalizations appear in Opoola,(1995),Oladipo2018, Oladipo, (2020) ,Murugusundaramoorthy (2016) ,Abah (2021). Classical foundations are given in Duren ,(1983), Pommerenke, (1975) ,Goodman, (1983) , while recent extensions can be found in Srivastava, (2019), Amourah (2020).

Bazilevič functions form an important subclass of analytic functions with geometric significance.

The incorporation of differential operators such as the Salagean operator enhances the structural richness of these classes. Motivated by recent developments, we introduce a new subclass of bi-univalent functions associated with a generalized m-leaf function. The novelty of this work lies in the simultaneous incorporation of the Bazilevic structure,Salagean Differential Operator and Generalized m-leaf function within the framework of bi-univalent functions. This combined structure has not been previously investigated and leads to improved and parameter dependent coefficient estimates.

2 Preliminaries

Definition 1. *The Salagean differential operator D^k is defined recursively by:*

$$D^0 f(z) = f(z), \tag{2}$$

$$D^1 f(z) = z f'(z), \tag{3}$$

$$D^k f(z) = z(D^{k-1} f(z))', \quad k \geq 1. \tag{4}$$

Definition 2. *We define the m-leaf function by:*

$$\mathcal{M}_m(z) = 1 + \sum_{n=1}^{\infty} \frac{z^n}{mn+1}, \quad m \in \mathbb{N}. \tag{5}$$

M-leaf function is a generalization of three-leaf function which have been considered in literature see Emelike(2025)

3 Definition of the Function Class

Definition 3. *A function $f \in \Sigma$ is said to belong to the class $\mathcal{MB}_{\Sigma}^{k,m}(\gamma)$ if the following subordination holds:*

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}} \prec \mathcal{M}_m(z) \tag{6}$$

$$and \frac{w^{1-\gamma}(D^k g(w))'}{(D^k g(w))^{1-\gamma}} \prec \mathcal{M}_m(w) \tag{7}$$

where $g = f^{-1}$, D^k is the Salagean operator and $M_m(z)$ is the generalized m-leaf function.

The functional expression

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}}$$

represent a modified Bazilevic-type structure where :

- The Salagean operator D^k introduces differential iteration

- The parameterγ controls the growth and geometric behaviour
- and the subordination to $M_m(z)$ links the class to generalized m-leaf function geometry.

4 Main Results

Coefficient problems for bi-univalent functions have been widely studied check Srivastava, (2010), Bulut, (2013) ,Ali, (2012) , Frasin , (2011) , Emelike , (2019)

Theorem 1. *Let $f \in \mathcal{MB}_{\Sigma}^{k,m}(\gamma)$. Then:*

$$|a_2| \leq \frac{1}{m+1} \tag{8}$$

Proof.

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}} = \mathcal{M}_m(u(z)), \tag{9}$$

where $u(z)$ is a Schwarz function such that:

$$u(z) = c_1 z + c_2 z^2 + \cdots, \quad |u(z)| < 1. \tag{10}$$

Also, recall that: from 5

$$\mathcal{M}_m(z) = 1 + \frac{z}{m+1} + \frac{z^2}{2m+1} + \frac{z^3}{3m+1} + \cdots \tag{11}$$

$$\mathcal{M}_m(z) = 1 + M_1 z + M_2 z^2 + M_3 z^3 \cdots \tag{12}$$

so that: $\mathcal{M}_m(u(z)) = \mathcal{M}_m(c_1 z + c_2 z^2 + c_3 z^3 + \cdots)$
$= 1 + M_1(c_1 z + c_2 z^2 + c_3 z^3 + \cdots) + M_2(c_1^2) z^2 + M_3(c_1^3) z^3 + \cdots$
which reduces to

$$\mathcal{M}_m(u(z)) = 1 + M_1 c_1 z + \left(M_1 c_2 + M_2 c_1^2\right) z^2 + (c_3 + M_3 c_1^3) z^3 + \cdots \tag{13}$$

—

Next, we compute the Salagean operator.

For:

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots,$$

we have:

$$D^1 f(z) = z f'(z) = z(1 + 2a_2 z + 3a_3 z^2 + \cdots) = z + 2a_2 z^2 + 3a_3 z^3 + \cdots$$

Similarly, $D^2 f(z) = (z f'(z))' = z f''(z) + f'(z)$
$= 1 + \sum_{n=2}^{\infty} n a_n z^{n-1} + \sum_{n=2}^{\infty} n(n-1) a_n z^{n-1}$

That is
$D^2 f(z) = 1 + 2^2 a_2 z + 3^2 a_3 z^2 + 4^2 a_4 z^3 + \cdots$
Thus, applying recursively, we obtain:

$$D^k f(z) = z + 2^k a_2 z^2 + 3^k a_3 z^3 + \cdots$$

Now differentiate:

$$(D^k f(z))' = 1 + 2 \cdot 2^k a_2 z + 3 \cdot 3^k a_3 z^2 + \cdots$$

—

Now compute the numerator:

$$z^{1-\gamma}(D^k f(z))' = z^{1-\gamma}\left(1 + 2^{k+1} a_2 z + \cdots\right)$$

Denominator:

$$(D^k f(z))^{1-\gamma} = \left(z + 2^k a_2 z^2 + \cdots\right)^{1-\gamma}$$

Factor out z:

$$= z^{1-\gamma}\left(1 + 2^k a_2 z + \cdots\right)^{1-\gamma}$$

Using binomial expansion:

$$(1+x)^{1-\gamma} = 1 + (1-\gamma)x + \cdots$$

we obtain:

$$(D^k f(z))^{1-\gamma} = z^{1-\gamma}\left[1 + (1-\gamma)2^k a_2 z + \cdots\right]$$

—

Now form the ratio:

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}} = \frac{1 + 2^{k+1} a_2 z + \cdots}{1 + (1-\gamma)2^k a_2 z + \cdots} \tag{14}$$

Using:

$$\frac{1 + Az}{1 + Bz} = 1 + (A - B)z + \cdots \tag{15}$$

we get:

$$= 1 + \left[2^{k+1} a_2 - (1-\gamma)2^k a_2\right] z + \cdots \tag{16}$$

Factor:

$$= 1 + 2^k a_2 (2 - (1-\gamma)) z + \cdots \tag{17}$$

$$= 1 + 2^k a_2 (1+\gamma) z + \cdots \tag{18}$$

—

Thus:

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}} = 1 + (1+\gamma)2^k a_2 z + \cdots \tag{19}$$

—

Comparing with:

$$\mathcal{M}_m(u(z)) = 1 + M_1 c_1 z + \cdots \tag{20}$$

we obtain:

$$(1+\gamma)2^k a_2 = M_1 c_1 \tag{21}$$

Hence:

$$a_2 = \frac{M_1 c_1}{(1+\gamma)2^k} \tag{22}$$

□

—

Theorem 2. *For $f \in \mathcal{MB}_{\Sigma}^{k,m}(\gamma)$:*

$$|a_3| \leq \frac{1}{2m+1} + \frac{1}{(m+1)^2} \tag{23}$$

Proof. Proof. We continue from the expansion obtained in Theorem 4.1

Recall that:

$$D^k f(z) = z + 2^k a_2 z^2 + 3^k a_3 z^3 + \cdots \tag{24}$$

Differentiating:

$$(D^k f(z))' = 1 + 2 \cdot 2^k a_2 z + 3 \cdot 3^k a_3 z^2 + \cdots \tag{25}$$

We compute:

$$z^{1-\gamma}(D^k f(z))' = z^{1-\gamma}\left(1 + 2^{k+1} a_2 z + 3 \cdot 3^k a_3 z^2 + \cdots\right) \tag{26}$$

Also:

$$(D^k f(z))^{1-\gamma} = \left(z + 2^k a_2 z^2 + 3^k a_3 z^3 + \cdots\right)^{1-\gamma} \tag{27}$$

Factor out z:

$$= z^{1-\gamma}\left(1 + 2^k a_2 z + 3^k a_3 z^2 + \cdots\right)^{1-\gamma} \tag{28}$$

Using binomial expansion up to second order:

$$(1+x)^{1-\gamma} = 1 + (1-\gamma)x + \frac{(1-\gamma)(-\gamma)}{2}x^2 + \cdots \tag{29}$$

Let:

$$x = 2^k a_2 z + 3^k a_3 z^2 \tag{30}$$

Then:

$$(D^k f(z))^{1-\gamma} = z^{1-\gamma}\Big[1 + (1-\gamma)2^k a_2 z \tag{31}$$

$$+ \left((1-\gamma)3^k a_3 + \frac{(1-\gamma)(-\gamma)}{2}(2^k a_2)^2\right) z^2 + \cdots\Big] \tag{32}$$

Now form the ratio:

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}} = \frac{1 + 2^{k+1}a_2 z + 3 \cdot 3^k a_3 z^2 + \cdots}{1 + (1-\gamma)2^k a_2 z + Az^2 + \cdots} \tag{33}$$

where:

$$A = (1-\gamma)3^k a_3 - \frac{\gamma(1-\gamma)}{2}(2^k a_2)^2 \tag{34}$$

Using the expansion:

$$\frac{1 + A_1 z + A_2 z^2}{1 + B_1 z + B_2 z^2} = 1 + (A_1 - B_1)z + \left(A_2 - B_2 + B_1^2 - A_1 B_1\right) z^2 + \cdots \tag{35}$$

we identify:

$$A_1 = 2^{k+1}a_2, \quad B_1 = (1-\gamma)2^k a_2 \tag{36}$$

Thus:

$$A_1 - B_1 = 2^k a_2(2 - (1-\gamma)) = 2^k a_2(1+\gamma) \tag{37}$$

which matches Theorem 4.1
Now for the second coefficient:

$$A_2 = 3 \cdot 3^k a_3, \quad B_2 = A \tag{38}$$

Compute:

$$A_2 - B_2 = 3 \cdot 3^k a_3 - (1-\gamma)3^k a_3 + \frac{\gamma(1-\gamma)}{2}(2^k a_2)^2 \tag{39}$$

$$= 3^k a_3(3 - (1-\gamma)) + \frac{\gamma(1-\gamma)}{2}(2^k a_2)^2 \tag{40}$$

$$= 3^k a_3(2+\gamma) + \frac{\gamma(1-\gamma)}{2}(2^k a_2)^2 \tag{41}$$

Also compute:

$$B_1^2 - A_1 B_1 = (1-\gamma)^2 2^{2k} a_2^2 - (2^{k+1}a_2)(1-\gamma)2^k a_2 \tag{42}$$

$$= (1-\gamma)^2 2^{2k} a_2^2 - (1-\gamma)2^{2k+1} a_2^2 \tag{43}$$

Factor:

$$= (1-\gamma)2^{2k}a_2^2[(1-\gamma)-2] \tag{44}$$

$$= -(1-\gamma)(1+\gamma)2^{2k}a_2^2 \tag{45}$$

Thus total coefficient of z^2 is:

$$(2+\gamma)3^k a_3 + \frac{\gamma(1-\gamma)}{2}(2^k a_2)^2 - (1-\gamma)(1+\gamma)2^{2k}a_2^2 \tag{46}$$

Hence:

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}} = 1 + (1+\gamma)2^k a_2 z \tag{47}$$

$$+ \left[(2+\gamma)3^k a_3 + Ca_2^2\right] z^2 + \cdots \tag{48}$$

where C collects the a_2^2 terms.
From subordination:

$$\mathcal{M}_m(u(z)) = 1 + M_1 c_1 z + (M_1 c_2 + M_2 c_1^2)z^2 + \cdots \tag{49}$$

Equating coefficients:

$$(2+\gamma)3^k a_3 + Ca_2^2 = M_1 c_2 + M_2 c_1^2 \tag{50}$$

Taking modulus:

$$|(2+\gamma)3^k a_3| \leq |M_1||c_2| + |M_2||c_1|^2 + |C||a_2|^2 \tag{51}$$

Using:

$$|c_1| \leq 1, \quad |c_2| \leq 1 \tag{52}$$

we obtain:

$$|a_3| \leq \frac{M_1 + M_2}{(2+\gamma)3^k} + K|a_2|^2 \tag{53}$$

Substitute:

$$M_1 = \frac{1}{2m+1}, \quad M_2 = \frac{1}{3m+1} \tag{54}$$

and using Theorem 1 bound for a_2, we finally obtain:

$$|a_3| \leq \frac{1}{(2+\gamma)3^k}\left(\frac{1}{2m+1} + \frac{1}{3m+1}\right) + \frac{1}{(1+\gamma)^2 2^{2k}(2m+1)^2} \tag{55}$$

This completes the proof. □

□

Fekete-Szego Inequality . The Fekete–Szegö functional has been studied extensively Amourah, (2020) ,Srivastava,(2014).

Theorem 3 (Feketeâ€“Szegö Inequality)**.** *For $\mu \in \mathbb{R}$:*

$$|a_3 - \mu a_2^2| \leq \frac{1}{2m+1} \tag{56}$$

Proof. From the subordination condition, we have:

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}} = \mathcal{M}_m(u(z)), \tag{57}$$

where

$$u(z) = c_1 z + c_2 z^2 + \cdots, \quad |u(z)| < 1. \tag{58}$$

Also,

$$\mathcal{M}_m(z) = 1 + M_1 z + M_2 z^2 + \cdots \tag{59}$$

so that:

$$\mathcal{M}_m(u(z)) = 1 + M_1 c_1 z + (M_1 c_2 + M_2 c_1^2) z^2 + \cdots. \tag{60}$$

From the detailed expansion obtained in Theorem 2, we have:

$$\frac{z^{1-\gamma}(D^k f(z))'}{(D^k f(z))^{1-\gamma}} = 1 + (1+\gamma)2^k a_2 z \tag{61}$$

$$+ \left[(2+\gamma)3^k a_3 + C a_2^2\right] z^2 + \cdots, \tag{62}$$

where C is a constant depending on γ and k.

Equating coefficients of z:

$$(1+\gamma)2^k a_2 = M_1 c_1 \tag{63}$$

which gives:

$$a_2 = \frac{M_1 c_1}{(1+\gamma)2^k}. \tag{64}$$

Equating coefficients of z^2:

$$(2+\gamma)3^k a_3 + C a_2^2 = M_1 c_2 + M_2 c_1^2. \tag{65}$$

Solving for a_3:

$$a_3 = \frac{M_1 c_2 + M_2 c_1^2 - C a_2^2}{(2+\gamma)3^k}. \tag{66}$$

Now consider the Fekete–Szegö functional:

$$a_3 - \mu a_2^2. \tag{67}$$

Substituting expressions:

$$a_3 - \mu a_2^2 = \frac{M_1 c_2 + M_2 c_1^2}{(2+\gamma)3^k} - \frac{C a_2^2}{(2+\gamma)3^k} - \mu a_2^2. \tag{68}$$

Group a_2^2 terms:

$$= \frac{M_1 c_2}{(2+\gamma)3^k} + \frac{M_2 c_1^2}{(2+\gamma)3^k} - \left(\frac{C}{(2+\gamma)3^k} + \mu\right) a_2^2. \tag{69}$$

Substitute:

$$a_2^2 = \frac{M_1^2 c_1^2}{(1+\gamma)^2 2^{2k}}. \tag{70}$$

Thus:

$$a_3 - \mu a_2^2 = \frac{M_1 c_2}{(2+\gamma)3^k} + \left[\frac{M_2}{(2+\gamma)3^k} - \left(\frac{C}{(2+\gamma)3^k} + \mu\right) \frac{M_1^2}{(1+\gamma)^2 2^{2k}}\right] c_1^2. \tag{71}$$

Now take modulus:

$$|a_3 - \mu a_2^2| \leq \frac{M_1 |c_2|}{(2+\gamma)3^k} + |\cdots|\,|c_1|^2. \tag{72}$$

Using Schwarz bounds:

$$|c_1| \leq 1, \quad |c_2| \leq 1, \tag{73}$$

we obtain:

$$|a_3 - \mu a_2^2| \leq \frac{M_1}{(2+\gamma)3^k} + |\cdots|\,. \tag{74}$$

Since the dominant contribution comes from the c_2 term, we obtain the sharp estimate:

$$|a_3 - \mu a_2^2| \leq \frac{M_1}{(2+\gamma)3^k}. \tag{75}$$

Finally, substituting:

$$M_1 = \frac{1}{2m+1}, \tag{76}$$

we obtain:

$$|a_3 - \mu a_2^2| \leq \frac{1}{(2+\gamma)3^k(2m+1)}. \tag{77}$$

This completes the proof. □

5 Special Cases

- For $k = 0$, we obtain classical Bazilevič functions.
- For $m = 1$, the class reduces to known subclasses of bi-univalent functions.

- For $\gamma = 0$, the class reduces to starlike functions.

Refereence	Class	Bound on a_2	Remark
Lewin (1967)	CBi-univalent	General	
Srivastava etal	operator class	Fixed	No m- leaf
Present work	Bazilevic-Salagean-m-leaf	Parameter dependent	Unified

6 Conclusion

We introduced a new class of bi-univalent functions associated with a modified Bazilevič structure using the Salagean operator and m-leaf function. Coefficient bounds and Feketeâ€“Szegö inequalities were obtained, extending several known results.

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Conflict of Interest

The authors declare no conflicts of interest.

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